A university level online course in algebra with automatic assessment system

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1. EXECUTIVE SUMMARY

In the first half of 2005, the authors began their work on designing and creating online courses in mathematics, aimed at covering the curricula of higher technical schools. The authors' primary goal was to combine extensive lecture notes material with a large collection of highly interactive exercises, in order to give a student as much room for practice as possible.

1.1. Background

The first e-course that the authors created throughout 2005 (with later enhancements and extensions) was the course in algebra with analytical geometry. The material is delivered in the form of Web pages supplemented with interactive exercises and math simulators. Over 200 types of math problems are supported in the form of Java-driven tests, varying from simple, drill exercises to sophisticated problems that involve graphing or performing specific transformations in the exercise window. Moreover, the exercises are not static. Each test is capable of repeatedly generating problems with randomly selected data. A teacher can flexibly configure a test to meet specific needs (such as the difficulty level, the method of grading, the time constraint for the solution etc). The student has access to a step-by-step presentation of the correct solution. Individual exercises can be combined into comprehensive exams that in turn are used both for practice and as an element of the administrative grading system.

The authors have been employing the e-course in everyday teaching for 7 semesters now. Not only is traditional in-class instruction accompanied by a complete set of materials on the Internet. The whole grading system of the course can be based on automatic exams.

In the fall semester 2007, a number of computer terminal rooms have been set up in Wroclaw University of Technology for the sole purpose of automatic assessment. As of this writing, the number of students that took or are passing the course in that way is close to 2000.

1.2. Conclusions

The combination of traditional instruction with highly automated control of the learners' progress has been giving results that by far outgrew the authors' original expectations. Thanks to practically unbounded possibilities to practice, with a large number of problem types, the student is offered highly innovative learning environment. This has strikingly positive impact on students' own motivation to study. The final grades together with the students' feedback, gathered at the end of each semester, leave no doubt as to the effectiveness of the new system. Additionally, the automation of many standard exercises enables the teacher to make a better use of the classroom meetings, in the sense that more time is available for the discussion of more difficult problems and for applications.

2. MOTIVATION AND CONCEPTUAL FRAMEWORK

Typically, when a portion of math material is being taught, the student is first familiarized with given math concepts or methods followed (or accompanied) by suitable examples, then come math problems that the student is supposed to work out by themselves. In few other areas does problem-solving play such an important role in the student's progress as in mathematics, and in few other areas is the student's active attitude towards independent work as crucial.

Automatic testing in math have been drawing the attention of math instructors' and e-learning content developers for many years. There exist some truly amazing solutions in that field, to mention the famous ALEKS (see [Cosyn]). (In fact, it was ALEKS that gave the authors most of the motivation to develop the e-course in algebra).

Most of learning management systems (LMS) include general assessment tools, such as online quizzes. From the point of view of math instruction, however, the use of such tools is limited to simple problems. Math problems are rarely limited to finding a numerical value. In many cases the solution involves graphing or providing an expression. Besides, every LMS-based assessment tool has the downside that it is bound to a specific LMS. No reusability is thus easily available.

The solution of an individual math problem usually breaks down into two or more steps (except, of course, simplistic problems involving the application of a single correctly identified formula or rule). When grading a math exam, the teacher typically devises a grading procedure for each individual problem in such a way that each part of the solution is assigned a given fraction of total score for the graded problem. In that way, the grading procedure reflects the solution process: the student that arrives at a partial solution earns the corresponding partial credit. It is desirable that automatic assessment should take such partial solution into account.

From the student's point of view, there are other important elements in the learning process, like the availability of the correct solution, for example. Traditional textbooks typically provide correct answers to all (or a part of) exercises. Hints or full solutions are limited to selected problems only. Electronic assessment content should offer extensive measures of such kind of help.

3. E-COURSE – MAIN FEATURES

During the first 9 months of 2005, the authors designed and created an e-course in algebra with analytical geometry. The presence of highly interactive exercises and tests was the authors' main goal. The math content of the course is logically divided into shareable content objects (SCO), so the material can be easily aggregated into various organizations and rearranged, as needed. In everyday practice, a SCORM or an IMS package is being generated from the raw content, and then imported to an LMS (see references for SCORM). Note that the e-course is completely independent of any specific e-learning platform.

The math content covers complex numbers, polynomials, matrices and linear systems, analytical geometry in the plane (including conic sections) and analytical geometry in space. The material is offered in the form of Web pages supplemented with interactive exercises and simulators. Over 200 types of algebra problems are supported in the form of tests, varying from simple, drill-type exercises to quite sophisticated graphing problems or problems where the user is supposed to demonstrate mastery of a given method. Individual problems can be combined into comprehensive exams that can be used both for practice and as an element of the official grading procedures. The flexibility of automatic tests makes it possible to have the administrative grading system totally based on automatic assessment.

4. TECHNOLOGY OF AUTOMATIC ASSESSMENT

In order to transform a math problem into an interactive computer-driven entity the authors considered the notion of a *problem type*. By this the authors mean a math problem stated with a certain amount of generality in terms of specific problem data. In that convention, "Find the roots of the given polynomial" is an example of a problem type. An individual problem type gives rise to infinitely many specific problems with varying data. Obviously, there exist problems of substantially varying difficulty levels within the same problem type. As said above, a problem can be given to a student as an exercise or as a part of a more comprehensive exam.

4.1. Functionality of a single exercise

In single-exercise case, each individual problem type works as a Java applet that is capable of repeatedly offering the student a (theoretically) unbounded sequence of problems of that type. This happens each time that the student presses the **New test** button in the applet window. The student is required to solve the problem by hand and then enter the result(s) in the applet window through specially designed user interface elements: edit fields, spin controls, check boxes or radio buttons. The designer can additionally impose a time constraint on every test, forcing the student to provide the solution within prescribed amount of time.

The supported problem types are not limited to the situation where the solution is a numerical value or a set of numerical values. In fairly many problems, the solution is supposed to be provided as a drawing (e.g. the graph of an equation). In such cases, the student is provided with dedicated graphing tools in the applet window. The applet itself is designed in such a way that the student's graphical input can be formally checked for correctness.

There are many problems in matrix theory and the theory of systems of linear equations, where the didactic value implies computational complexity. More specifically, a problem is either trivial because it addresses a simple two- or a three-dimensional case, or is inevitably tedious due to a large amount of hand computations. In automatic exercises of that type, a student is given special tools for designing the desired matrix operations, the computational part being left to the program.

The "completeness check" feature is used, warning the student if the input is incomplete (e.g., some of edit fields are left blank). Typically, the student is given as many chances to complete the missing entries as he or she wishes (unless the time assigned for the solution is over).

After the student provides the solution in the applet window and presses the Submit button, the student's result is verified for correctness and graded. Different grading schemes are available, from simple "incorrect – correct" to numerical scores with partial credit supported.

The student has access to the correct solution through the Solution button in the applet window. The solution can be presented "all-at-once" or in steps. The solution always addresses the actual problem that the student was given, not a problem type.

Figure 1 illustrates the look of the applet window that offers exercises on solving linear systems by Jordan-Gauss elimination. Note the presence of matrix operations tools in the applet window.

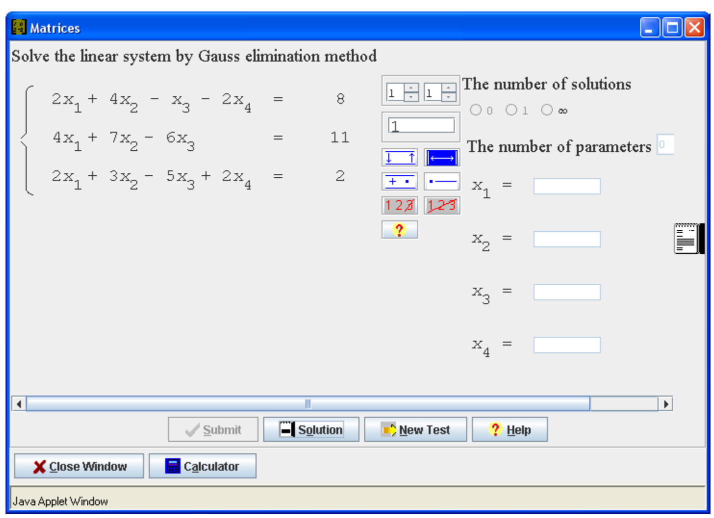

Figure 1. A single-problem applet window with matrix operations tools

4.2. Functionality of an exam

Individual problem types can be combined to produce more comprehensive exams. An exam can consist of a variable number of different problems, and each problem can be assigned a different score. A specific set of exam problems is generated based on a designer's settings in a configuration file (without the necessity of intervention in the source code). Since the data for each problem is randomly generated, no two different students receive identical problems. Moreover, the designer can either predefine the exact type of each problem on the exam, or to specify the probability with which problems of given types will appear. In that way, the distribution of problem types within an exam is further diversified.

From the student's perspective, an exam again works as an applet window, in which a collection of problems is given in the form of tabs (see Figure 2).

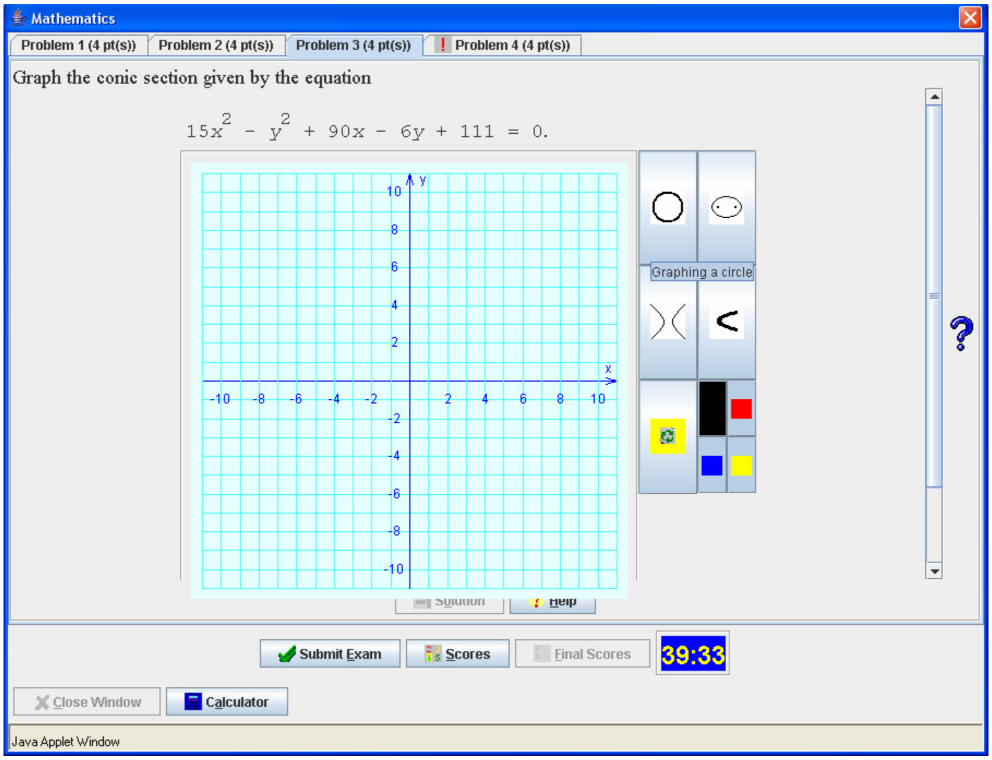

Figure 2. An exam applet window

The total score for the exam is available to the student as soon as the exam ends. Also, the student has immediate access to step-by-step solutions of all the exam problems.

When the student submits the exam for evaluation, all the solutions are initially verified for their correctness. The student is then informed about those problems whose solutions are not fully correct. The number of tries to correct the wrong solution depends on the teacher. In practice exams, the number of tries granted by the teacher is usually 2 or 3, while in administrative exams one try is usually granted.

The use of exams can be twofold. During the learning process, an exam serves as a practice utility, enabling the student to carry out self-evaluations. In that case, the student can launch an exam as many times as he or she wishes, getting different problem sets each time. Besides, the exams can be (and are) used as an element of the administrative grading system. In that case, the student's results are written to a database, with all necessary security and encryption conditions satisfied.

5. AUTOMATIC ASSESSMENT IN PRACTICE

In the spring semester 2005, the system of automatic assessment was applied to a pilot group of students. At that time, the course content was being created and made available to the students parallel to the in-class instruction. A relatively small group of 55 students was chosen due to limited availability of computer labs for the purpose of the experiment. Two mid-term exams and the final exam were given in the labs.

The teaching results and the students' feedback after the first semester were so promising that the authors decided to think of a larger-scale implementation. A few other math instructors were invited to join the grading system based on automatic assessment. It should be noted that standard classroom teaching was not reduced in any way. As heavily as the grading system relied on automatic exams, traditional assessment elements were retained, i.e. the students were earning part of the credit through classical paper tests.

The table below presents the numbers of students of Wroclaw University of Technology that were taking part in algebra courses with automatic assessment in consecutive semesters. The large differences between student numbers in spring and fall semesters result from the fact that algebra courses are typically taught in fall semesters. In spring semesters, make-up courses are offered with considerably smaller enrollment.

Table 1. Numbers of students in automatic assessment

Semester	Number of courses	Number of students
Spring 2005	1	55
Fall 2005	3	380
Spring 2006	2	95
Fall 2006	8	950
Fall 2007	7	900
Spring 2008	1	170

5.1. Organizational barriers

One of obvious advantages of automatic assessment is the fact that students can be tested more frequently at no additional cost of the teacher's work (grading the exams). Accordingly, the course material can be divided into smaller portions that the students are tested on, thus making the study more comfortable.

On the other hand, the expansion of automatic assessment has natural barriers. Assume that a student is to be tested 4 times during a semester (the number that seems reasonable from the didactic viewpoint). An individual exam usually includes 4 to 5 problems to be solved during one hour. A typical single lecture group consists of about 100 students. This yields a demand of 400 hours of computer terminal availability during a semester for every single lecture group of students. Since a typical computer lab consists of 30 terminals, the actual number of hours that the lab has to be available for automatic testing is about 13 hours per semester per a lecture group of students. As small as the latter number may appear, it should be kept in mind that existing computer labs are normally fully taken for instruction, and that the student's own class schedule has to be taken into account to avoid time conflicts.

Taking an automatic exam requires seamless functioning of all the involved technology, like a reliable e-learning LMS and database connectivity. To prevent the students from technology-related cheating, safety measures have to be taken, e.g., disabling the student's access to the Internet. All this requires extra organizational and technical effort. Technical support on the part of specially trained staff should be also ensured. All of that means extra costs.

5.2. Partially and fully proctored testing

With the above-mentioned issues in mind, and yet wishing to considerably enlarge the number of student groups that automatic testing would cover, the authors decided to employ a mixed grading system, where part of the testing would be proctored, and the other part would be distant. In the fall semesters 2005 and 2006 as well as the spring semester 2006 (see the table above for the numbers of students), the following grading procedures were assumed:

- Each student is taking 6 automatic exams from home; the lowest score (or a missed exam) is dropped; in that way, distant exams play the role of traditional homework assignments.
- At the end of the semester, each student takes a comprehensive final exam in a computer lab. To pass the course, the student needs to earn a predefined minimum score on that final exam; otherwise the student fails and is then given the standard second chance on the make-up exam.
- Provided that the student did sufficiently well on the final exam, the final grade is calculated based on the results of the final exam combined with a specified percentage of home-earned points. Additionally, the student can earn extra credit for in-class activity.

As the success of automatic testing became evident, Wroclaw University of Technology set up a number of new computer labs designed exclusively to serve for the purpose of automatic assessment. A small team of auxiliary staff has been trained to proctor the exams. Additional security measures were taken. The computer terminals had access to a dedicated small local network only, thus practically eliminating the risk of "technology-based cheating" on the students' part. The exam time schedules were designed at the beginning of the semester.

Starting the fall semester 2007, the following grading system was introduced:

- Each student is taking 5 automatic exams in a computer lab in proctored conditions; the lowest score (or a missed exam) is dropped.
- Additionally, the student can earn additional credit for in-class activity.
- The final grade is determined based on a defined score/grade conversion.
- If the rules above do not yield a passing grade, the student takes an extra comprehensive exam in a computer lab.

6. RESULTS AND STUDENTS' FEEDBACK

The teaching schedule, in which the student's progress is primarily measured through a series of automatic exams, proved very effective. The diagram below shows the distribution of final grades for the whole population of the so-called Basic Study Program (i.e. first-year students that are not yet assigned to specific engineering departments).

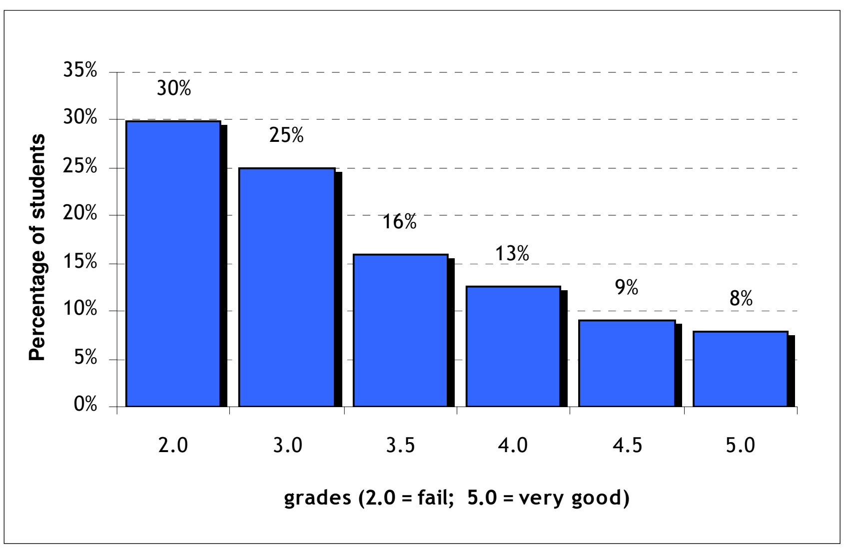

Figure 3. Final grades (Fall 2007, 480 students)

Similar results are repeatedly obtained for other groups and other semesters. With math courses usually having failure rate over 50%, the results are clear evidence of the advantage of the new system.

After each semester, student evaluations are carried out. The students are usually asked to evaluate two factors: the effectiveness and the fairness of the system. They are also offered to provide written comments.

During all the seven semesters that the system has been in use, the student evaluations are very high. The unlimited opportunity to practice is one of most commonly emphasized element. Two diagrams below show the distribution of student responses to two main survey questions for a group of over 370 students. The evaluation applies to the fall semester 2007, when all the automatic exams were proctored. The students were responding by giving a grade from 1 (lowest) to 6 (highest).

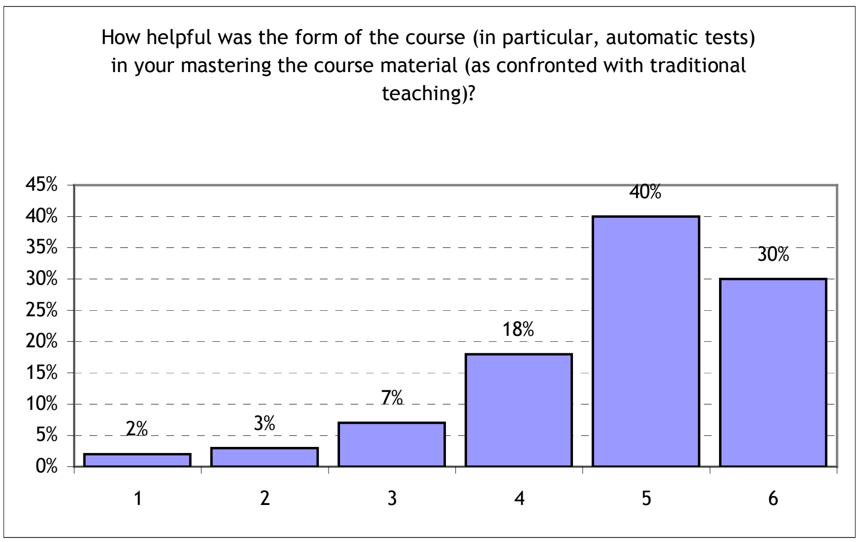

Figure 4. Student evaluations (effectiveness)

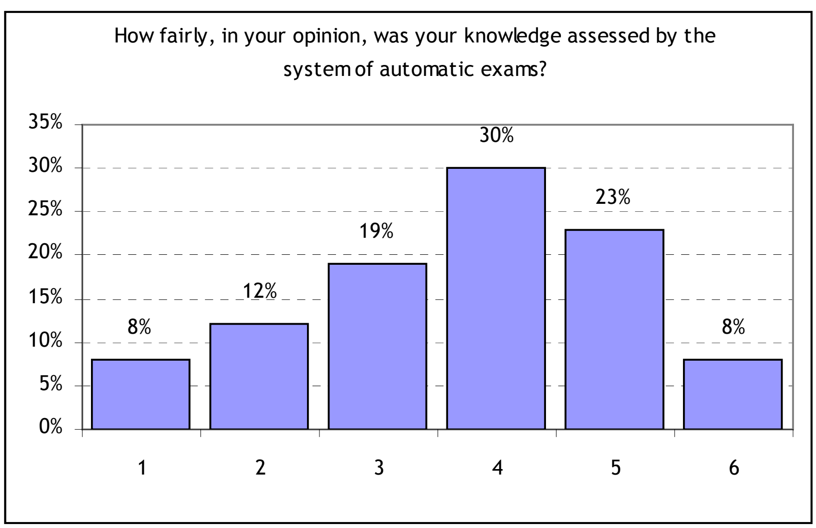

Figure 5. Student evaluations (fairness)

7. PRESENT AND FUTURE PROJECTS

As soon as the algebra e-course was given for the first time, demand arose for other math courses to be supported in a similar way. The students' readiness to sign up for math courses with automatic assessment system (expressed in student surveys) was going far beyond the authors' expectations. It was thus obvious that the development should proceed in the same direction.

As of this writing, the following e-courses are at the design stage:

- Mathematical Analysis I (calculus of 1 variable)
- Linear Algebra with Elements of Abstract Algebra (2nd author)
- Remedial Mathematics (aimed at those students who need to improve their skills in elementary mathematics).

In the case of Mathematical Analysis course, CAS software (webMathematica) is combined with Java technology to support symbolic calculations that are part of many calculus problems. In the spring semester 2008, a group of 180 students are in the Mathematical Analysis I e-course, with the grading system partially relying on automatic assessment (two electronic mid-term exams followed by two traditional mid-term exams).

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